A Mathematical Model for the Behavior of Thrust Reversers

H. Y. Chang[*]
San Diego State College, San Diego, Calif.

AND

J. P. Waidelich[†]
Rohr Corporation, Chula Vista, Calif.

The flow within an external target type thrust reverser is analyzed, using the simplifying assumptions that the flow is inviscid, incompressible, and two dimensional. The analysis is essentially an application of the freestreamline theory with numerical evaluation of integrals. In the analysis, a pair of transformations were used which allow the ejection angle to be expressed as a function of reverser geometric variables in two equations which have the form of improper integrals. These equations are then solved by numerical integration, with an approximation technique used at the singular points. Results are presented in the form of graphs which show the jet exit angle ϕ as a function of the reverser geometric variables: L/d, H/d, sweep angle, and endplate angle. Excellent agreement with test results from a two-dimensional water jet is shown.

Nomenclature

A,B,C,E,F,P	= points or angles on the transformation planes
a,b,c,e,f,p	= coordinates in t planes corresponding to points A,B,C,E,F,P, respectively
C_1,C_2	= const
D	= jet diameter of the cross section at entrance to fluid freebody
D_N	= nozzle diameter
d	= half height of rectangular jet cross section at entrance to fluid freebody
H	= endplate height
L	= door length
q	= velocity magnitude
r	= integer
t	= variable of integration; abscissa in t plane
U	= variable of integration
u	= velocity component in horizontal direction
V	= undisturbed reference velocity
v	= velocity component in vertical direction
w	= complex function
x, y	= variables; $1/(t-a)$, $1/(t-b)$, respectively
z	= complex coordinate in z plane
$\angle E$	= endplate angle
$\angle S$	= door sweep angle
α_1,α_2	= angles of inclination
ζ	= derivative of complex velocity potential
θ	= inclination of velocity vector
μ,ν	= dimensionless angles: α_2/π, α_1/π, respectively
ϕ	= ejection angle
Ω	= complex coordinate in Ω plane

1. Introduction

A THRUST reverser can be considered as a segmental deflector (see Figs. 1 and 2) which reverses the exhaust gases of a jet engine so as to decelerate the airplane. In the study of the performance of thrust reversers, the ejection angle of the reversed jet ϕ is the dependent variable. It is necessary to determine the ejection angle for any thrust reverser since, from this angle, the reverser effectiveness (defined as the ratio of the reversed thrust to forward thrust) is calculated. In addition, the shape of the plume formed by the interaction of the reversed jet and the freestream flow is also influenced by the ejection angle. The plume, depending on its shape, may distort the flowfield near the airframe surface and affect airplane stability.

To the authors' knowledge, a theoretical study of the impingment of a free jet on a reverser with four independent variables has never been made.[1,2] A similar study was made by Sarpkaya[1] but was restricted to a reverser with zero sweep angle and 90° endplate angle. With two additional variables, the mathematical analysis shows a far greater complexity.

The actual jet flow during normal operation is circular in cross section. During a reverse mode, a simplified two-dimensional flow is assumed, since the doors themselves are essentially two dimensional. Potential flow is assumed, i.e., compressible and viscous effects are neglected.

The freestream-line theory has been used to calculate the ejection angle as a function of the reverser door geometry, which is described by the door length L, the endplate height H, the door sweep angle $\angle S$, and the endplate angle $\angle E$. Hence,

$$\phi = f_1(L/d, H/d, \angle S, \angle E) \tag{1}$$

where d represents a linear dimension (half height) of the two-dimensional jet. The resulting equations are in the form of improper trigonometric integrals which were evaluated by

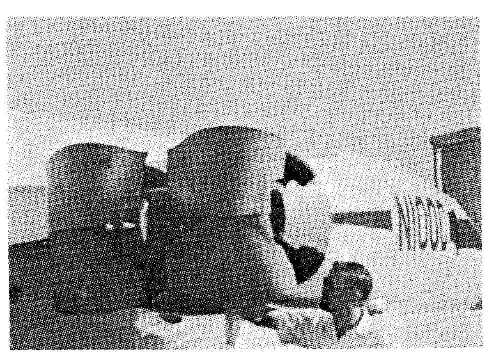

Fig. 1 Target thrust reverser.

Presented as Paper 69-3 at the AIAA 7th Aerospace Sciences Meeting, New York, January 20–22, 1969; submitted January 22, 1969; revision received August 14, 1969. This research was supported by Rohr Corporation, under an Independent Research and Development Authorization.

* Assistant Professor, Department of Aerospace Engineering. Member AIAA.

† Chief of Engineering Sciences, Member AIAA.

using a fourth-order Runge-Kutta numerical method, plus an approximation technique to evaluate the integrals near singular points.

2. Analysis

The following is the mathematical basis for the flow pattern inside the thrust reverser. From the flow analysis, the ejection angle can be expressed quantitatively as a function of the reverser door geometry which includes the door length, the endplate height, the door sweep angle, and the endplate angle. The analysis follows the direct application of complex variables and conformal mapping theory as it applies to free-streamline flows. For more detailed information, the reader may refer to standard texts.[3,4]

The two-dimensional flow region under consideration for a thrust reverser is shown in Fig. 3a. The effect of freestream flow is assumed to be small when establishing the jet exit angle. This appears to be true since, in practice, cowls or other forebodies create a wake within which most of the reverser flow occurs. The nonviscous flow assumption requires the flow to be uniform at the nozzle and at the exhaust jet. Also, Bernoulli's equation requires that the flow velocity be constant at the freestream line (since pressure is constant at the free surface). In order to determine the mathematical flow pattern inside the reverser, a pair of transformations are employed to transform the flow region from the z plane (physical plane) to the Ω plane and the t plane (see Fig. 3). Now, $\zeta = dw/dz$ results in

$$\zeta = u - iv = qe^{-i\theta} \tag{2}$$

in which u and v are velocity components in the horizontal and vertical directions, respectively, q and θ are the magnitude and inclination of the velocity vector at a certain point.

The equation

$$\Omega = \ln(V/\zeta) = \ln(V/q) + i\theta \tag{3}$$

maps the flow region of the z plane onto the indicated region of the Ω plane. In the Ω plane, points of unit velocity ($q = V = 1$) are on the vertical axis, stagnation points A and B with zero velocities ($q = 0$) are at infinity in the direction of the horizontal axis.

The Schwartz-Christoffel transformation is used to map the boundary in the Ω plane onto the real axis of the t plane and the closed region in the Ω plane onto the upper half of the t plane. The flow is from point E to point F; its mathematical expression can be determined easily in the t plane by the superposition of a point source at E and a point sink at F. In the t plane, the scale of representation between e and c is chosen to be unity and f is at infinity.

From the Schwartz-Christoffel transformation, we have

$$d\Omega/dt = \text{const}\,(t-p)(t-a)(t-b)[(t-e)(t-c)]^{1/2} \tag{4}$$

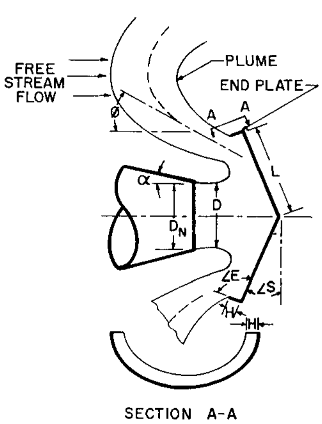

Fig. 2 Thrust reverser nomenclature.

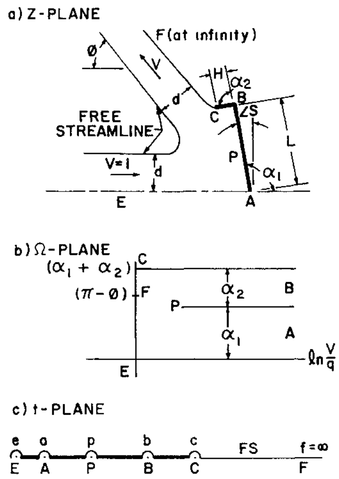

Fig. 3 Conformal transformation planes.

where $t = p$, a, b, e, and c represent the values of t at points P, A, B, E, and C, respectively. As this equation shows, singularities are at $t = a$, b, e, and c. They are represented by semicircular indentations in the t plane. Separating Eq. (4) by partial fractions, we have

$$\frac{d\Omega}{dt} = \text{const}\,\frac{b-p}{b-a}\,\frac{1}{(t-b)[(t-e)(t-c)]^{1/2}} + \text{const}\,\frac{a-p}{a-b}\,\frac{1}{(t-a)[(t-e)(t-c)]^{1/2}}$$

Letting $(t - b) = 1/y$ in one case and $(t - a) = 1/x$ in the other and integrating, this becomes

$$\Omega = \text{const}\int\frac{dy}{[(b-e)(b-c)y^2 + (2b-e-c)y + 1]^{1/2}} + \text{const}\int\frac{dx}{[(a-e)(a-c)x^2 + (2a-e-c)x + 1]^{1/2}} = C_1\ln\frac{[(t-e)(c-b)]^{1/2} + [(t-c)(e-b)]^{1/2}}{(t-b)^{1/2}} + C_2\ln\frac{[(t-e)(c-a)]1/2 + [(t-c)(e-a)]^{1/2}}{(t-a)^{1/2}} \tag{5}$$

where C_1 and C_2 are constants which will now be evaluated.

From Eqs. (3) and (5), we have

$$C_1\ln\frac{[(t-e)(c-b)]^{1/2} + [(t-c)(e-b)]^{1/2}}{(t-b)^{1/2}} + C_2\ln\frac{[(t-e)(c-a)]^{1/2} + [(t-c)(e-a)]^{1/2}}{(t-a)^{1/2}} = \ln\frac{V}{q} + i\theta$$

Referring to Fig. 3, with $c > b > a > e$; it is evident that, when $c > t > b$, the left-hand side of the previous equation is real, hence $\theta = 0$.

This is along BC or the endplate. Again, when $b > t > a$, the part with coefficient C_2 is real, whereas the other is imaginary and equals $C_1 \ln(1/i)$. Therefore

$$\theta = C_1\ln(1/i) = -C_1\pi/2 \tag{6}$$

along AB or the door.

Finally, when $a > t > e$, both parts of the left-hand side of the previous equation are imaginary, and we have

$$\theta = (C_1 + C_2)\ln(1/i) = -(C_1 + C_2)\pi/2 \tag{7}$$

along EA.

Referring to Fig. 4, we see that if $\theta = 0$ along BC, the values of θ along BA and AE are $-\alpha_2$ and $-(\alpha_2 + \alpha_1)$, respectively.

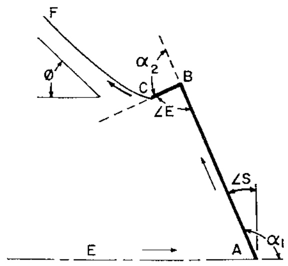

Fig. 4 Angle definition.

Therefore, from Eqs. (6) and (7) we obtain $C_1 = 2\alpha_2/\pi = 2\mu$ and $C_2 = 2\alpha_1/\pi = 2\nu$.

The constants of Eq. (5) thus have been evaluated. Consequently,

$$\Omega = 2\mu \ln \frac{[(t-e)(c-b)]^{1/2} + [(t-c)(e-b)]^{1/2}}{(t-b)^{1/2}} + 2\nu \ln \frac{[(t-e)(c-a)]^{1/2} + [(t-c)(e-a)]^{1/2}}{(t-a)^{1/2}} \quad (8)$$

The scale of representation has been chosen with $c - e = 1$. Then, for $c > t > e$, there is a real angle U between zero and $\pi/2$, such that $c - t = \cos^2 U$ and $t - e = \sin^2 U$. This gives us 1) $t - b = \sin^2 U - \sin^2 B = \sin(U + B)\sin(U - B)$, etc., and 2) $[(t-e)(c-b)]^{1/2} + [(t-c)(e-b)]^{1/2} = \sin U \cos B + \cos U \sin B = \sin(U + B)$, etc. Therefore, Eq. (8) can be put in the following form:

$$\Omega = \mu \ln[\sin(U+B)/\sin(U-B)] + \nu \ln[\sin(U+A)/\sin(U-A)] \quad (9)$$

Ejection Angle

This is the angle the reversed jet makes with the forward jet. From Fig. 4, ϕ is the ejection angle and corresponds to the inclination θ of point F at infinity. Letting $t = \infty$, Eq. (8) becomes

$$\Omega = 2\mu \ln[(c-b)^{1/2} + (e-b)]^{1/2} + 2\nu \ln[(c-a)^{1/2} + (e-a)]^{1/2} = 2\mu \ln(\cos B + i \sin B) + 2\nu \ln(\cos A + i \sin A) = 2\mu B i + 2\nu A i = \ln(V/q) + i\theta$$

Therefore, $\theta = 2\mu B + 2\nu A$. This is the angle between BC and CF. Hence the ejection angle becomes

$$\phi = 2\mu B + 2\nu A - \angle S + \angle E - \pi/2 \quad (10)$$

Door Length and Endplate Height

The door length and endplate height are given by

$$\int_a^b \left|\frac{dz}{dt}\right| dt \text{ and } \int_b^c \left|\frac{dz}{dt}\right| dt$$

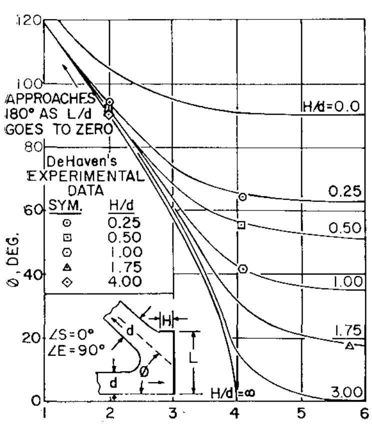

Fig. 5 Ejection angle vs reverser door geometry with $\angle S = 0°$ and $\angle E = 90°$ as determined by Sarpkaya (Ref. 1); experimental data by DeHaven (Ref. 1).

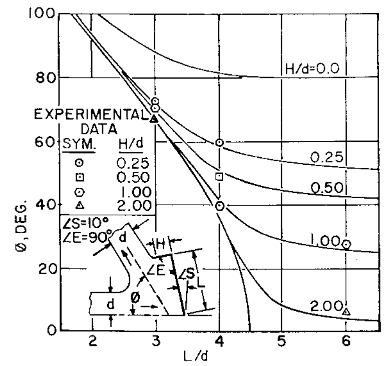

Fig. 6 Ejection angle vs reverser door geometry with $\angle S = 10°$ and $\angle E = 90°$.

Considering the latter, it can be written in the form

$$\int_B^{\pi/2} \left|\frac{dz}{dU}\right| dU$$

From the source and equal sink at E and F, the following complex potential is obtained for the upper half of the t plane:

$$w = (Vd/\pi)[\ln(t-e) - \ln(t-f)] \quad (11)$$

Now since $\zeta = dw/dz$ and $f = \infty$, hence,

$$dz = \frac{1}{\zeta}\frac{dw}{dt}dt = \frac{d}{\pi}\frac{V}{\zeta}\left(\frac{1}{t-e} - \frac{1}{t-f}\right)dt = \frac{d}{\pi}\frac{V}{\zeta}\frac{dt}{(t-e)} = \frac{d}{\pi}\frac{V}{\zeta}\frac{d(\sin^2 U)}{\sin^2 U} = \frac{2d}{\pi}\frac{V}{\zeta}\cot U\, dU \quad (12)$$

and since, from Eq. (3)

$$V/\zeta = e^{\Omega}$$

Therefore, from Eq. (9)

$$V/\zeta = [\sin(U+B)/\sin(U-B)]^{\mu} \times [\sin(U+A)/\sin(U-A)]^{\nu} \quad (13)$$

From Eqs. (12) and (13), we have

$$dz = 2d/\pi[\sin(U+B)/\sin(U-B)]^{\mu} \times [\sin(U+A)/\sin(U-A)]^{\nu} \cot U\, dU$$

For the door length

$$\frac{L}{d} = \int_A^B \frac{2}{\pi}\left[\frac{\sin(U+B)}{\sin(B-U)}\right]^{\mu}\left[\frac{\sin(U+A)}{\sin(U-A)}\right]^{\nu} \cot U\, dU \quad (14)$$

For the endplate height

$$\frac{H}{d} = \int_B^{\pi/2} \frac{2}{\pi}\left[\frac{\sin(U+B)}{\sin(U-B)}\right]^{\mu}\left[\frac{\sin(U+A)}{\sin(U-A)}\right]^{\nu} \cot U\, dU \quad (15)$$

Equations (14) and (15) have singularities at A and B, hence they are improper trigonometric integrals. They must be evaluated with reference to a path (as shown in Fig. 3) with semicircular indentations at each of these points. An approximation technique for evaluating the improper integrals is given in the appendix.

Table 1 Values of A and B used for computer program

A, rad						
0.001	0.01	0.05	0.10	0.15	0.20	0.25
0.30	0.35	0.40	0.45	0.50	0.55	0.60
0.65	0.70	0.75	0.80	0.90	1.00	1.20
B, rad						
0.05	0.10	0.15	0.20	0.25	0.30	0.35
0.40	0.50	0.60	0.70	0.80	0.90	0.95
1.00	1.05	1.10	1.15	1.20	1.25	1.30
1.35	1.40	1.45	1.50	1.56		

3. Solution to Equations

A computer program was written in FORTRAN IV for the IBM 360 computer.[5] For a reverser with a certain door sweep angle and endplate angle, this program computes the ejection angle ϕ, the door-length ratio L/d, and the endplate height ratio H/d, according to Eqs. (10), (14), and (15), respectively. Different combinations of A and B values were assigned to these equations to compute different sets of results for ϕ, L/d, and H/d. Table 1 shows the values of A and B used.

A Runge-Kutta fourth-order method[6] was used to evaluate the integrals for L/d and H/d with a length of interval for integration equal to 0.1. Since the Runge-Kutta method fails at singular points, it was used at a small distance (0.02 in our case) away from the singular points A and B in Eqs. (14) and (15). Within the distance of 0.02 around the singular points, the integrals were evaluated according to the approximation technique given in the Appendix. The selected values of computer results with $\angle S = 0°$ and $\angle E = 90°$ are tabulated in Table 2.

4. Results

The results are presented in Figs. 5, 6, and 7. In these figures, the ejection angle ϕ is expressed as a function of the four independent variables for which the door sweep angle $\angle S$ has two values (0° and 10°); the endplate angle $\angle E$ has two values (90° and 105°); the door length L/d and endplate height H/d have multiple values.

A comparison of Figs. 5 and 6 shows that an increase in $\angle S$ reduces the ejection angle. An inspection of Figs. 6 and 7 indicates that a greater $\angle E$ causes a further reduction of the ejection angle in the region of low L/d but it increases the ejection angle for larger values of L/d.

Figure 5 represents Sarpkaya's exact analysis. The numerical solutions derived herein predict the same results as those calculated by Sarpkaya (to within the resolution of Fig. 5). The experimental data of DeHaven as adopted by Sarpkaya are also reproduced on the figure.

DeHaven's data as reported by Sarpkaya were obtained from axisymmetric flow tests. Sarpkaya used these data to substantiate his theoretical two-dimensional results.

Experiments have been conducted at San Diego State College. Plastic models were tested in a flume which had a 6-in. width and the jet had a thickness d of 1.5 in. The water discharge was 86 gallons/min for all runs. Figure 8 shows the deflected jet with $\angle S = 10°$, $\angle E = 90°$, $L/d = 4$, and $H/d = 0.25$. It also shows how the ejection angle was measured. Experimental results are shown on Figs. 6 and 7 with the theoretical predictions. Again the results appear in good agreement.

Appendix

"Runge-Kutta Method" may be used to evaluate the integrals where they are applicable, but it is evident that it fails at

Table 2 Selected values of computer results with $S = 0°$, $E = 90°$

A	B	ϕ	L/d	H/d
0.001	0.05	29.2	4.00	3.35
0.001	0.65	37.3	5.34	0.95
0.001	1.30	74.5	4.84	0.08
0.15	0.25	22.9	3.88	3.16
0.15	0.65	45.8	3.69	1.11
0.15	1.30	83.1	2.87	0.09
0.30	0.35	37.2	3.53	3.78
0.30	0.65	54.4	3.27	1.32
0.30	1.30	91.7	2.27	0.09
0.60	0.65	71.6	2.58	2.61
0.60	1.30	108.9	1.46	0.10
0.70	0.95	94.5	1.87	0.75
0.70	1.30	114.6	1.24	0.10

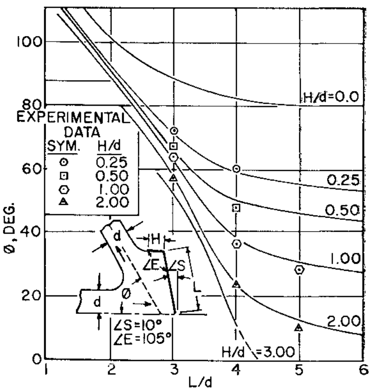

Fig. 7 Ejection angle vs reverser door geometry with $\angle S = 10°$ and $\angle E = 105°$.

a singular point. The following approximate method, which was originally developed by Bryan and Jones,[7] will consequently be used near the singular point. Consider

$$\int_B^{\pi/2} \left[\frac{\sin(U+B)}{\sin(U-B)}\right]^{\mu} \left[\frac{\sin(U+A)}{\sin(U-A)}\right]^{\nu} \cot U \, dU$$

where $U = B$ is the singular point. Set $U - B = x$, and the integrand becomes

$$\frac{1}{\sin^{\mu} x} \frac{\sin^{\mu}(2B + x)\sin^{\nu}(A + B + x)\cot(B + x)}{\sin^{\nu}(B - A + x)} = F(x)$$

When $x \to 0$ (i.e., $U \to B$), $\sin^{\mu} x \to x^{\mu}$, and we may write $F(x) = x^n \phi(x)$, where $n = -\mu$. $\phi(x)$ is now expanded in powers of x, being finite and continuous within the limits, and

$$F(x) = Ax^n + Bx^{n+1} + Cx^{n+2} + \ldots = y \quad \text{(A1)}$$

Let y_1 be the value of y $[=F(x)]$ when $x = h$

y_2 be the value of $y[=F(x)]$ when $x = 2h$

.

.

y_r be the value of $y[=F(x)]$ when $x = rh$

Suppose

$$\int_0^{rh} y dx \text{ (to be required)} = (P_1 y_1 + P_1 y_2 + \ldots + P_r y_r)h = P_1(Ah^n + Bh^{n+1} + Ch^{n+2} + \ldots)h + P_2[A(2h)^n + B(2h)^{n+1} + C(2h)^{n+2} + \ldots]h + \ldots P_r[A(rh)^n + B(rh)^{n+1} + C(rh)^{n+2} \ldots]h = (P_1 + 2^n P_2 + \ldots + r^n P_r)Ah^{n+1} + (P_1 + 2^{n+1}P_2 + \ldots + r^{h+1}P_r)Bh^{n+2} + \ldots$$

But if we integrate (A1), it becomes

$$\int_0^{rh} y dx = \frac{A(rh)^{n+1}}{n+1} + \frac{B(rh)^{n+2}}{n+2}$$

Therefore $P_1 + 2^n P_2 + \ldots + r^n P_r = r^{n+1}/(n+1)$, $P_1 + 2^{n+1}P_2 + \ldots + r^{n+1}P_r = r^{n+2}/n + 2$, and so on, which when solved, gives the values of P.

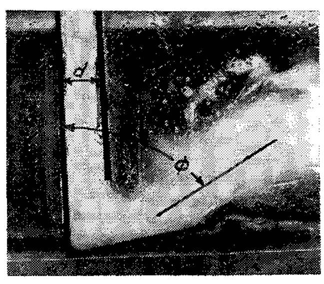

Fig. 8 Deflected jet showing the measurement of the ejection angle with $L/d = 4$, $H/d = 0.25$, $\angle S = 10°$, $\angle E = 90°$, and $\phi = 60°$.

In our case, the lengths have been calculated for different values of μ and ν, and r has been taken to be 2 giving then

$$P_1 = 2^{n+2}/(n + 1)(n + 2) = 2^{2-\mu}/(1 - \mu)(2 - \mu) \quad \text{(A2)}$$

$$P_2 = 2n/(n + 1)(n + 2) = -2\mu/(1 - \mu)(2 - \mu) \quad \text{(A3)}$$

and

$$\int_0^{2h} F(x)dx = (P_1y_1 + P_2y_2)h \quad \text{(A4)}$$

The formula derived here, then, will be used in the immediate vicinity of A and B, from B_1 to B say, along the door length; $0 < B_1 < B$; from A to A_1 along the door length; $0 < A < A_1$; and from B to B_2 along the endplate height; $0 < B < B_2$. The values of the integrals from A_1 to B_1 and from B_2 to $\pi/2$ will be evaluated according to the Runge-Kutta method.

To test the accuracy of this procedure, the integral

$$\int_0^a \frac{dx}{(a^2 - x^2)^{1/2}}$$

has been evaluated and found to be 1.55, as compared with the true value $\pi/2 = 1.57$. The error is, therefore, about 1.5% with these intervals.

5. References

[1] Sarpkaya, T., "Deflection of Jets—II. Symmetrically Placed U-Shaped Obstacle," *Studies in Engineering*, Vol. 35, State University of Iowa, Ames, Iowa, 1953.

[2] Tinney, E. R. et al., "Free Streamline Theory for Segmental Jet Deflectors," *Journal of the Hydraulic Division*, HY5, American Society of Civil Engineers, Sept. 1961.

[3] Robertson, J. M., *Hydrodynamics in Theory and Application*, Prentice-Hall, Englewood Cliffs, N. J., 1965.

[4] Streeter, V. L., *Fluid Dynamics*, McGraw-Hill, New York, 1948.

[5] Chang, H. Y., "Flow Analysis Inside Thrust Reversers," Engineering Rept. 24-2287, 1968, Rohr Corp., Chula Vista, Calif.

[6] Ralston, A. and Wilf, H. S., *Mathematical Methods for Digital Computers*, Wiley, New York, 1960.

[7] Bryan, G. H. and Jones, R., "Discontinuous Fluid Motion Past a Bent Plane, with Special Reference to Aeroplane Problems," *Proceedings of Royal Society of London*, Vol. A91, No. 354-70, 1915.